Arithmetic

Contents

Australian Business Number (ABN):
64 011 074 054

http://tutoring.div1.com.au
P.O. Box 1353, Meadowbank, N.S.W. 2114, AUSTRALIA

I. Addition

a) Table for positive quantities

In the following table the number in the top row is added to the number in the leftmost column. Example highlighted: 6 + 2 = 8 .
The result of addition is called the "sum".

Practise by

- reading through lines of the table below;
- writing out a copy of the table (while looking at it);
- completing blank versions of the table (without looking at it).

You can start with the smaller numbers (*e.g*. numbers from 1 to 10) if necessary, and then work up to larger numbers.

+	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	**2**	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	3	**4**	5	6	7	8	9	10	11	12	13	14	15	16	17
3	4	5	**6**	7	8	9	10	11	12	13	14	15	16	17	18
4	5	6	7	**8**	9	10	11	12	13	14	15	16	17	18	19
5	6	7	8	9	**10**	11	12	13	14	15	16	17	18	19	20
6	7	8	9	10	11	**12**	13	14	15	16	17	18	19	20	21
7	8	9	10	11	12	13	**14**	15	16	17	18	19	20	21	22
8	9	10	11	12	13	14	15	**16**	17	18	19	20	21	22	23
9	10	11	12	13	14	15	16	17	**18**	19	20	21	22	23	24
10	11	12	13	14	15	16	17	18	19	**20**	21	22	23	24	25
11	12	13	14	15	16	17	18	19	20	21	**22**	23	24	25	26
12	13	14	15	16	17	18	19	20	21	22	23	**24**	25	26	27
13	14	15	16	17	18	19	20	21	22	23	24	25	**26**	27	28
14	15	16	17	18	19	20	21	22	23	24	25	26	27	**28**	29
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	**30**

The table is symmetrical across the diagonal, because for addition the order of operations is not important. Thus, 3 + 9 = 9 + 3 , and 124 + 1 = 1 + 124 .
NOTE: This means that addition is a "commutative" operation.

Notice that the diagonal is where the given number is **doubled**.

b) Strategies

Relate more difficult sums to simpler ones that you are more confident in.

EXAMPLE: Calculate 29 + 8.

- OPTION A: An easier sum is 30 +8 = 38 . 29 is one less than 30. So the answer here should be one less than 38. Therefore 29 + 8 = 37 .
- OPTION B: We must add a total of 8 onto 29, but we can do it in stages. First we add 1 onto 29, yielding 29 + 1 = 30 (which is a convenient reference point). As we've now

added 1, we have a remaining task of adding 7 (because 8−1=7) onto our intermediate result, *i.e.* 30 + 7 = 37 . Therefore 29 + 8 = 37 .

- OPTION C: 29 can be split up into 20 and 9, so 29 + 8 = 20 + 9 + 8 . We can add the 8 and 9 to get 17, and then add this to 20: 29 + 8 = 20 + 9 + 8 = 20 + 17 = 37 .

EXAMPLE: Calculate 15 + 16.

- OPTION A: An easier sum is 15 +15 = 30 . 16 is one more than 15. So the answer here should be one more than 30. Therefore 15 + 16 = 31 .
- OPTION B: Split the sum up into ‘tens’ and ‘units’. 15 + 16 = 10 + 5 + 10 + 6. Now rearrange to obtain simpler sums: 15 + 16 = 10+10 + 5+6 = 20 + 11 = 31.

c) Table for positive & negative quantities

When dealing with negative quantities, the table can be thought of as including some subtraction operations. Construct a “number line”, in which positive numbers and changes are towards the right, whereas negative numbers and changes are to the left.

In the following table the number in the top row is added to the number in the leftmost column. Example highlighted: −2 + −6 = −8 .

Practise by

- reading through lines of the table below;
- completing blank versions of the table.

+	−7	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6	+7
−7	**−14**	−13	−12	−11	−10	−9	−8	−7	−6	−5	−4	−3	−2	−1	0
−6	−13	**−12**	−11	−10	−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1
−5	−12	−11	**−10**	−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2
−4	−11	−10	−9	**−8**	−7	−6	−5	−4	−3	−2	−1	0	1	2	3
−3	−10	−9	−8	−7	**−6**	−5	−4	−3	−2	−1	0	1	2	3	4
−2	−9	−8	−7	−6	−5	**−4**	−3	−2	−1	0	1	2	3	4	5
−1	−8	−7	−6	−5	−4	−3	**−2**	−1	0	1	2	3	4	5	6
0	−7	−6	−5	−4	−3	−2	−1	**0**	1	2	3	4	5	6	7
+1	−6	−5	−4	−3	−2	−1	0	1	**2**	3	4	5	6	7	8
+2	−5	−4	−3	−2	−1	0	1	2	3	**4**	5	6	7	8	9
+3	−4	−3	−2	−1	0	1	2	3	4	5	**6**	7	8	9	10
+4	−3	−2	−1	0	1	2	3	4	5	6	7	**8**	9	10	11
+5	−2	−1	0	1	2	3	4	5	6	7	8	9	**10**	11	12
+6	−1	0	1	2	3	4	5	6	7	8	9	10	11	**12**	13
+7	0	1	2	3	4	5	6	7	8	9	10	11	12	13	**14**

Once again the given number is **doubled** on the diagonal.
(On the diagonal running the other way, the two values cancel to give zero.)

Australian Business Number (ABN):
64 011 074 054

http://tutoring.div1.com.au
P.O. Box 1353, Meadowbank, N.S.W. 2114, AUSTRALIA

II. Subtraction

a) Table for positive quantities

In the following table the number in the top row is subtracted from the number in the leftmost column. Example highlighted: 6 – 2 = 4 .

NOTE: Here 6 is the "minuend" and 2 is the "subtrahend". The subtrahend is always taken away from the minuend.

Practise as for Addition. Construct of a "number line" to avoid confusion.

–	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11	−12	−13	−14
2	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11	−12	−13
3	2	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11	−12
4	3	2	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11
5	4	3	2	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10
6	5	4	3	2	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9
7	6	5	4	3	2	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8
8	7	6	5	4	3	2	1	**0**	−1	−2	−3	−4	−5	−6	−7
9	8	7	6	5	4	3	2	1	**0**	−1	−2	−3	−4	−5	−6
10	9	8	7	6	5	4	3	2	1	**0**	−1	−2	−3	−4	−5
11	10	9	8	7	6	5	4	3	2	1	**0**	−1	−2	−3	−4
12	11	10	9	8	7	6	5	4	3	2	1	**0**	−1	−2	−3
13	12	11	10	9	8	7	6	5	4	3	2	1	**0**	−1	−2
14	13	12	11	10	9	8	7	6	5	4	3	2	1	**0**	−1
15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	**0**

On the diagonal the given numbers **cancel out**.

Notice that the table is not perfectly symmetrical across the diagonal, although there is a similarity: the numbers in the upper-right triangle are negative versions of those in the lower-left triangle.

For subtraction the order of operations is very important: 20 – 5 = 15 , but 5 – 20 = −15 .

NOTE: This means that subtraction is <u>not</u> a "commutative" operation.

Observe that although 20 – 5 ≠ 5 – 20 , the answers differ 'only' in terms of their sign, *i.e.* by the presence or absence of a 'negative' sign.

If two numbers are the same when their signs (positive or negative) are disregarded, we say that they have the same "magnitude" or "absolute value".

b) Strategies

i. Ignore some digits

Temporarily ignore 'thousands' *etc.* when subtracting small numbers from large ones.

EXAMPLE: Calculate 5329 – 8.

The 5329 can be split up as 5320 + 9, so 5329 – 8 = 5320 + 9 – 8 . Now 9 – 8 is simply 1, so 5329 – 8 = 5320 + 9 – 8 = 5320 +1 = 5321 .

Australian Business Number (ABN): 64 011 074 054
http://tutoring.div1.com.au
P.O. Box 1353, Meadowbank, N.S.W. 2114, Australia

ii. Treat as addition

Try recasting the problem as an addition when the result is expected to be smallish.

EXAMPLE: Calculate 5329 − 5118.

Adding 2 to 5118 would take us up to 5120 (use this as a reference point). Add another 9 to get up to 5129. Now the only difference is in the 'hundreds' column, so add 200 to 5129 to reach 5329. This tells us that the difference between 5118 and 5329 is 2+9+200 = 211 , so 5329 − 5118 = 211 .

EXAMPLE: Calculate 5129 − 4997.

- OPTION A: Adding 3 to 4997 would take it up to 5000 (use this as a reference point). Adding a further 129 to 5000 gets us to 5129. Thus the difference between 5129 and 4997 must be 3+129, which is 132 (add one to 129, then a further two). Hence 5129 − 4997 = 132 .
- OPTION B: Another way to think about it is to relate this question to a simpler calculation. 5129 − 5000 is an easier calculation: the answer to that is just 129. 5000 is three more than 4997, so when we subtract 4997 we are not removing as much as in a subtraction of 5000, and so there should be more remaining; specifically, the amount remaining after subtraction of 4997 should be three more than for subtraction of 5000. Three more than 129 is 132. Thus 5129 − 4997 = 132 .
- OPTION C: As before, start with a simpler calculation: 5129 − 5000 = 129. 5000 is chosen as a convenient reference point. Now we need to continue to get down to 4997, *i.e.* find 5000 − 4997 = 3 . Thus 5129 − 4997 = (5129 − 5000) + (5000 − 4997) = 129 + 3 = 132 .

Notice that the two "5000s" cancel out in the above equation, so our choice of reference point won't affect the final result. It is just that some numbers are easier to work with than others.

iii. Swap order

If a negative result is expected — such as when subtracting large numbers from small ones — then consider temporarily swapping the order of the numbers to obtain an intermediate result.

EXAMPLE: Calculate 25 − 35.

The number being subtracted (35) is larger than the number being subtracted from (25), so a negative result should be expected.

NOTE: Here 25 is the "minuend" and 35 is the "subtrahend". The subtrahend is always taken away from the minuend.

It is simpler for us to think about the opposite calculation, namely 35 − 25 = 10. Or, less formally, we may simply observe that the difference between 25 and 35 is 10.

Given this result, and knowing that a negative answer is required, we can conclude that 25 − 35 = −10.

The sign of a result can be confirmed by plotting the initial quantity and the various changes on a number line.

c) Table for positive & negative quantities

In the following table the number in the top row is subtracted from the number in the leftmost column. Example highlighted: −2 − −6 = 4 .

NOTE: Here −2 is the "minuend" and −6 is the "subtrahend". The subtrahend is always taken away from the minuend.

Practise as above.

Australian Business Number (ABN): 64 011 074 054

http://tutoring.div1.com.au

P.O. Box 1353, Meadowbank, N.S.W. 2114, AUSTRALIA

−	−7	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6	+7
−7	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11	−12	−13	−14
−6	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11	−12	−13
−5	2	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11	−12
−4	3	2	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11
−3	4	3	2	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10
−2	5	4	3	2	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8	−9
−1	6	5	4	3	2	1	**0**	−1	−2	−3	−4	−5	−6	−7	−8
0	7	6	5	4	3	2	1	**0**	−1	−2	−3	−4	−5	−6	−7
+1	8	7	6	5	4	3	2	1	**0**	−1	−2	−3	−4	−5	−6
+2	9	8	7	6	5	4	3	2	1	**0**	−1	−2	−3	−4	−5
+3	10	9	8	7	6	5	4	3	2	1	**0**	−1	−2	−3	−4
+4	11	10	9	8	7	6	5	4	3	2	1	**0**	−1	−2	−3
+5	12	11	10	9	8	7	6	5	4	3	2	1	**0**	−1	−2
+6	13	12	11	10	9	8	7	6	5	4	3	2	1	**0**	−1
+7	14	13	12	11	10	9	8	7	6	5	4	3	2	1	**0**

In the lower left corner of the table, negative numbers are subtracted from positive numbers. This is the same as adding the positive version of the subtrahend, and so the results are all positive. For example, $+3 - -4 = +3 + +4 = +7$.

Recall the rule that two identical signs combine to make a "plus", while two dissimilar signs combine to make a "minus". CAUTION: This rule only applies to signs undergoing *multiplication* or *division*; in the above calculation, the two negative signs are *adjacent*, which implies multiplication of the two signs.

Using a number line we would start 3 units to the right, and move "−4 units to the left" (to the left because of the subtraction), which is equivalent to moving +4 units to the right, ending up 7 units to the right (on the positive side).

In the upper right corner of the table, positive numbers are subtracted from negative numbers, making the result 'even more negative'. For example, $-6 - +3 = -9$.

Another way to think about this is to factorise the expression to remove a common factor of −1. That is, $-6 - +3 = -1\times(6+3) = -1\times(9) = -9$. This matches how we would figure out the result using a number line: start 6 units to the left, and go a further 3 units to the left, ending up 9 units to the left (on the negative side).

In the other corners (upper left and lower right), the sign of the result depends on the relative magnitude of the two quantities in the subtraction.

III. Multiplication

a) Table for positive quantities

In the following table the number in the top row multiplies the number in the leftmost column. Example highlighted: $6 \times 2 = 12$.

The result of multiplication is called the "product".

Results printed in feint text are infrequently encountered, and need not be memorised. (Although you should still be capable of computing them.)

When practicing, firstly work on mastering multiplication of smaller numbers, such as the "ten times" table (numbers from 1 to 10), then work up to larger numbers. Look for patterns.

Australian Business Number (ABN):
64 011 074 054

http://tutoring.div1.com.au
P.O. Box 1353, Meadowbank, N.S.W. 2114, AUSTRALIA

×	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	**1**	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	2	**4**	6	8	10	12	14	16	18	20	22	24	26	28	30
3	3	6	**9**	12	15	18	21	24	27	30	33	36	39	42	45
4	4	8	12	**16**	20	24	28	32	36	40	44	48	52	56	60
5	5	10	15	20	**25**	30	35	40	45	50	55	60	65	70	75
6	6	12	18	24	30	**36**	42	48	54	60	66	72	78	84	90
7	7	14	21	28	35	42	**49**	56	63	70	77	84	91	98	105
8	8	16	24	32	40	48	56	**64**	72	80	88	96	104	112	120
9	9	18	27	36	45	54	63	72	**81**	90	99	108	117	126	135
10	10	20	30	40	50	60	70	80	90	**100**	110	120	130	140	150
11	11	22	33	44	55	66	77	88	99	110	**121**	132	143	154	165
12	12	24	36	48	60	72	84	96	108	120	132	**144**	156	168	180
13	13	26	39	52	65	78	91	104	117	130	143	156	**169**	182	195
14	14	28	42	56	70	84	98	112	126	140	154	168	182	**196**	210
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	**225**

The table is symmetrical across the diagonal, because for multiplication the order of operations is not important. Thus, $3 \times 9 = 9 \times 3$, and $12 \times 5 = 5 \times 12$.

NOTE: This means that multiplication is a "commutative" operation.

Notice that the diagonal is where the given number is **squared**.

b) Table for positive & negative quantities

In the following table the number in the top row multiplies the number in the leftmost column. Example highlighted: $-2 \times -6 = +12$.

×	−7	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6	+7
−7	**49**	42	35	28	21	14	7	0	−7	−14	−21	−28	−35	−42	−49
−6	42	**36**	30	24	18	12	6	0	−6	−12	−18	−24	−30	−36	−42
−5	35	30	**25**	20	15	10	5	0	−5	−10	−15	−20	−25	−30	−35
−4	28	24	20	**16**	12	8	4	0	−4	−8	−12	−16	−20	−24	−28
−3	21	18	15	12	**9**	6	3	0	−3	−6	−9	−12	−15	−18	−21
−2	14	12	10	8	6	**4**	2	0	−2	−4	−6	−8	−10	−12	−14
−1	7	6	5	4	3	2	**1**	0	−1	−2	−3	−4	−5	−6	−7
0	0	0	0	0	0	0	0	**0**	0	0	0	0	0	0	0
+1	−7	−6	−5	−4	−3	−2	−1	0	**1**	2	3	4	5	6	7
+2	−14	−12	−10	−8	−6	−4	−2	0	2	**4**	6	8	10	12	14
+3	−21	−18	−15	−12	−9	−6	−3	0	3	6	**9**	12	15	18	21
+4	−28	−24	−20	−16	−12	−8	−4	0	4	8	12	**16**	20	24	28
+5	−35	−30	−25	−20	−15	−10	−5	0	5	10	15	20	**25**	30	35
+6	−42	−36	−30	−24	−18	−12	−6	0	6	12	18	24	30	**36**	42
+7	−49	−42	−35	−28	−21	−14	−7	0	7	14	21	28	35	42	**49**

Australian Business Number (ABN): 64 011 074 054

http://tutoring.div1.com.au

P.O. Box 1353, Meadowbank, N.S.W. 2114, AUSTRALIA

In the upper left and lower right corners of the table, the results are all positive. For example, $-3 \times -4 = +12$ and likewise $+3 \times +4 = +12$.

Recall the rule that two identical signs combine to make a "plus", while two dissimilar signs combine to make a "minus".

In the lower left and upper right corners of the table, the results are all negative. For example, $+3 \times -4 = -12$ and likewise $-3 \times +4 = -12$.

c) Strategies

i. Relate to simpler expressions

EXAMPLE: Calculate 12 × 8.

11 × 8 might be easier to remember; it is 88. 12 is one more than 11, so the result must have one additional lot of 8 added to it. 88 + 8 = 96 (88+2=90, and a further 6 to get to 96, given that 2+6=8). Hence, $12 \times 8 = 96$.

EXAMPLE: Calculate 9 × 7.

10 × 7 is easier to remember; it is 70. 9 is one less than 10, so the result must have one fewer lots of 7 added to it. 70 − 7 = 63. Hence, $9 \times 7 = 63$.

EXAMPLE: Calculate 15 × 12.

This involves larger numbers, but by thinking strategically it can be evaluated with mental arithmetic.

- OPTION A: Notice that multiplying by 12 is the same as doubling and then multiplying by six (because 12=2×6). So $15 \times 12 = 15 \times 2 \times 6$. Doubling 15 is easy; the result is 30. So now $15 \times 12 = 30 \times 6$. To multiply 30 by 6, recall that $3 \times 6 = 18$, and here 30 is ten times bigger than 3, so our result must be ten times bigger than 18 (just "add a zero"). Thus, $15 \times 12 = 180$.
- OPTION B: We could also have broken the 12 down into 4×3. This is also convenient because knowing that there are 60 minutes in an hour, and a quarter of an hour is 15 minutes, we quickly realise that four lots of 15 must be 60. So $15 \times 12 = 15 \times 4 \times 3 = 60 \times 3$. As before we temporarily ignore the zero and recognise that $6 \times 3 = 18$, and so $60 \times 3 = 180$. Thus, $15 \times 12 = 180$, as before.

ii. Deal with signs separately

EXAMPLE: Calculate −12 × −8.

Temporarily ignore the signs. From our previous discussion (or from memory), we know that $12 \times 8 = 96$.

Consider the signs. Here we have two "minus" signs being multiplied; multiplication of two identical signs must result in a "plus".

Combining these two conclusions, therefore $-12 \times -8 = +96$.

EXAMPLE: Calculate +9 × −7.

Temporarily ignore the signs. From our previous discussion (or from memory), we know that $9 \times 7 = 63$.

Consider the signs. Here we have one "plus" sign and one "minus" sign being multiplied; multiplication of two dissimilar signs must result in a "minus".

Combining these two conclusions, therefore $+9 \times -7 = -63$.

EXAMPLE: Calculate +3 × −3 × −7.

Here there are three quantities. What to do?

- OPTION A: Work pairwise. Firstly, $+3 \times -3$: we know that $3\times3=9$, and the result of this must be made negative, so $+3 \times -3 = -9$. Now we can simplify $+3 \times -3 \times -7$ to -9×-7 . As before, $9 \times 7 = 63$, but now the result of this must be made positive. Hence $+3 \times -3 \times -7 = +63$.
- OPTION B: Temporarily ignore the signs. $3\times3\times7 = 9\times7 = 63$. Now consider the signs. We have one “plus” sign and two “minus” signs being multiplied. The two “minus” signs ‘cancel each other out’, so this is equivalent to two “plus” signs, which is in turn equivalent to one “plus” sign. Hence $+3 \times -3 \times -7 = +63$.

Multiplication involving any EVEN number of “MINUS” signs will ‘cancel out’ to yield a single “PLUS” sign.
Multiplication involving any ODD number of “MINUS” signs will not ‘cancel out’, yielding a single “MINUS” sign.
This also works for division, or combinations of multiplication and division.

CAUTION: Do not apply this rule to expressions where the signed numbers are being added or subtracted!

IV. Powers

a) Table for positive quantities

In the following table the number in the leftmost column is raised to the power of the number in the top row. Example highlighted: $5^2 = 25$.

NOTE: Here 2 is the “index” and 5 is the “base”. The base is always raised to the power specified by the index.

Recall that a power of two represents the number multiplied by itself, or “squared”.
Thus, $4^2 = 4 \times 4 = 16$.

A number raised to the power of three is said to be “cubed”, and it is multiplied one more time than when it is squared.
Thus, $4^3 = 4 \times 4 \times 4 = 64$.

^	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	4	8	16	32	64	128	256	512	1024	2048	4096
3	3	9	27	81	243	729						
4	4	16	64	256	1024	4096						
5	5	25	125	625								
6	6	36	216									
7	7	49	343									
8	8	64	512	4096								
9	9	81	729									
10	10	100	1000	10000	100000	1000000	10 million	100 million	1 billion	10 billion	100 billion	1 trillion
11	11	121										
12	12	144										
13	13	169										
14	14	196										
15	15	225										
16	16	256	4096									

The symbol “^” is not formally recognised in mathematics as representing a power. However, it is a common convention in computer applications.

Australian Business Number (ABN):
64 011 074 054
http://tutoring.div1.com.au
P.O. Box 1353, Meadowbank, N.S.W. 2114, Australia

Results printed in feint text are infrequently encountered, and need not be memorised — although you should still be capable of computing them.

For example, 3^5 can be obtained by recognising that $3^5 = 3 \times 3^4 = 3 \times (3^2)^2 = 3 \times 9^2 = 3 \times 81$; then by mental arithmetic $3{\times}1{=}3$ and $3{\times}8{=}24$, so $3{\times}81{=}243$; hence $3^5 = 243$.

On the other hand, 7^3 would be computed as $7{\times}7^2{=}7{\times}49$. 49 is an inconvenient number to work with, but it is only one less than 50. $7{\times}50{=}350$. The result we seek should therefore be $7{\times}1{=}7$ less than 350. Hence $7^3 = 243$.

b) Index laws

The table above demonstrates some of the important index laws.

Firstly, when any number is raised to the power of 1, the number is unchanged. For example, $12^1 = 12$.

This means that we can imagine an 'invisible' index of 1 attached to any number, in the same way that we can interpret any positive number as having an 'invisible' "plus" sign in front of it.

Secondly, when 1 is raised to any power the result is still 1. For example, $1^{53} = 1$.

Thirdly, some results appear in several places in the table. For instance, 4096 appears four times (underlined):

$2^{12} = 4096$

$4^6 = 4096$

$8^4 = 4096$

$16^3 = 4096$

It is not immediately obvious why this should be so.

The pattern becomes more evident when we convert to a consistent base. Recall that $4{=}2^2$, $8{=}2^3$, and $16{=}2^4$. So the above equations can also be written:

$(2^1)^{12} = 2^{12} = 4096 = (2)^{12} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$(2^2)^6 = 4^6 = 4096 = (2{\times}2)^6 = (2{\times}2){\times}(2{\times}2){\times}(2{\times}2){\times}(2{\times}2){\times}(2{\times}2){\times}(2{\times}2)$

$(2^3)^4 = 8^4 = 4096 = (2{\times}2{\times}2)^4 = (2{\times}2{\times}2){\times}(2{\times}2{\times}2){\times}(2{\times}2{\times}2){\times}(2{\times}2{\times}2)$

$(2^4)^3 = 16^3 = 4096 = (2{\times}2{\times}2{\times}2)^3 = (2{\times}2{\times}2{\times}2){\times}(2{\times}2{\times}2{\times}2){\times}(2{\times}2{\times}2{\times}2)$

Notice that $1{\times}12{=}12$, $2{\times}6{=}12$, $3{\times}4{=}12$, and $4{\times}3{=}12$.

The pairs of indices given in the left-hand expressions can be simplified by using their product as a single index. That is, $(2^4)^3 = 2^{4\times3} = 2^{12}$. Likewise, $(2^3)^4 = 2^{3\times4} = 2^{12}$.

NOTE: Recall that multiplication is commutative, so $(2^4)^3 = (2^3)^4$.

A further index law allows us to 'expand' $(2{\times}2{\times}2{\times}2)^3$ as $2^3{\times}2^3{\times}2^3{\times}2^3$. Likewise, $(2{\times}2{\times}2)^4$ as $2^4{\times}2^4{\times}2^4$.

Given that $2^6 = 64$, what power would we have to raise 64 to in order to get a result of 4096?

What other results appear more than once in the table? Why?

c) Table for positive & negative quantities

In the following table the number in the leftmost column is raised to the power of the number in the top row. Example highlighted: $(-2)^{-3} = 1/(-2)^3 = -1/8$.

^	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6
−6				−1/216	+1/36	−1/6	+1	−6	+36	−216			
−5			+1/625	−1/125	+1/25	−1/5	+1	−5	+25	−125	+625		
−4	+1/4096	−1/1024	+1/256	−1/64	+1/16	−1/4	+1	−4	+16	−64	+256	−1024	+4096
−3	+1/729	−1/243	+1/81	−1/27	+1/9	−1/3	+1	−3	+9	−27	+81	−243	+729
−2	+1/64	−1/32	+1/16	−1/8	+1/4	−1/2	+1	−2	+4	−8	+16	−32	+64
−1	+1	−1	+1	−1	+1	−1	+1	−1	+1	−1	+1	−1	+1
0	N/A	N/A	N/A	N/A	N/A	N/A	+1*	0	0	0	0	0	0
+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
+2	+1/64	+1/32	+1/16	+1/8	+1/4	+1/2	+1	+2	+4	+8	+16	+32	+64
+3	+1/729	+1/243	+1/81	+1/27	+1/9	+1/3	+1	+3	+9	+27	+81	+243	+729
+4	+1/4096	+1/1024	+1/256	+1/64	+1/16	+1/4	+1	+4	+16	+64	+256	+1024	+4096
+5			+1/625	+1/125	+1/25	+1/5	+1	+5	+25	+125	+625		
+6				+1/216	+1/36	+1/6	+1	+6	+36	+216			

* The value of 0^0 is strictly undefined, but there is a common convention to set the value of this expression to 1 for convenience.

When zero is raised to a negative power the result is undefined (and there is no convention to define any convenient value).

V. Division

There is no generic 'Division Table'. The best option is to use the Multiplication Table in reverse. For example, knowing that $7 \times 5 = 35$ means that $35 \div 5$ must be 7, and $35 \div 7$ must be 5.

In this operation the "dividend" is always divided by the "divisor". When division is interpreted as a fraction, the dividend is equivalent to the numerator (top number in a fraction), and the divisor is equivalent to the denominator (bottom number in a fraction). The result of multiplication is called the "quotient".

a) Divisibility of large numbers

There are a few convenient rules for identifying whether a number is divisible by certain quantities.

All even numbers are **divisible by 2**. Odd numbers are not.

In all numbers **divisible by 3**, the sum of their digits is also divisible by 3.

EXAMPLE: State whether 9987654321 is exactly divisible by 3.

The sum of the digits in 9987654321 is $9+9+8+7+6+5+4+3+2+1 = 54$. If 54 is divisible by 3, then 9987654321 must also be divisible by 3. The sum of the digits in 54 is $5+4 = 9$. As 9 is obviously divisible by 3, then 54 must also be divisible by 3, and so must 9987654321.

All numbers **divisible by 4** end in a two-digit combination (in the 'tens' and 'units' columns) that is also divisible by 4. So knowing that 24 is divisible by 4 immediately tells us that 124, 3624, 65424, and 15151321024 must all be divisible by 4 too.

All numbers **divisible by 5** end with either a 5 or a 0 (in the 'units' column).

All numbers **divisible by 10** end with a 0 (in the 'units' column).

For some larger divisors, the above rules can be combined.

For instance, all numbers **divisible by 12** must be divisible by both 3 and 4.

EXAMPLE: Find whether 9987654321 is exactly divisible by 12.

From the preceding example we know that 9987654321 is divisible by 3. The last two digits are "21". As 21 is not divisible by 4, 9987654321 must not be divisible by 4 either. Hence 9987654321 is not divisible by 12.

Notice that 3 and 4 are <u>not</u> factors of one another. The approach would need to be changed if they were.

For all numbers **divisible by 8** the result after halving must be divisible by 4.

EXAMPLE: Find whether 300020 is exactly divisible by 8.

We first use mental arithmetic to halve 300020. Half of 30 is 15, and half of 20 is 10. So half of 300020 is 150010. The last two digits of this result are "10". As 10 is not divisible by 4, 150010 must not be divisible by 4 either. Hence 300020 is not divisible by 8.

Notice that 2 <u>is</u> a factor of 4. Hence the approach here differs from the approach for divisibility by 12.
Observe that 300020 is divisible by both 2 (it is even) and 4 (because 20 is divisible by 4), and yet 300020 itself is not exactly divisible by 8.

All numbers **divisible by 100** end with 00 (in the 'tens' and 'units' columns). All numbers **divisible by 1000** end with 000 (in the 'hundreds', 'tens' and 'units' columns). And so on.

Australian Business Number (ABN): 64 011 074 054

http://tutoring.div1.com.au

P.O. Box 1353, Meadowbank, N.S.W. 2114, AUSTRALIA

VI. Blank tables for practice

a) Addition

Table for positive quantities

In the following table the number in the top row is added to the number in the leftmost column. Example highlighted: 6 + 2 = 8 .

The result of addition is called the “sum”.

+	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1															
2															
3															
4															
5															
6		8													
7															
8															
9															
10															
11															
12															
13															
14															
15															

Table for positive & negative quantities

In the following table the number in the top row is added to the number in the leftmost column. Example highlighted: −2 + −6 = −8 .

Australian Business Number (ABN):
64 011 074 054

http://tutoring.div1.com.au
P.O. Box 1353, Meadowbank, N.S.W. 2114, AUSTRALIA

+	−7	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6	+7
−7															
−6															
−5															
−4															
−3															
−2		−8													
−1															
0															
+1															
+2															
+3															
+4															
+5															
+6															
+7															

b) Subtraction

Table for positive quantities

In the following table the number in the top row is subtracted from the number in the leftmost column. Example highlighted: 6 – 2 = 4 .

NOTE: Here 6 is the “minuend” and 2 is the “subtrahend”. The subtrahend is always taken away from the minuend.

−	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1															
2															
3															
4															
5															
6		4													
7															
8															
9															
10															
11															
12															
13															
14															
15															

Table for positive & negative quantities

In the following table the number in the top row is subtracted from the number in the leftmost column. Example highlighted: –2 – –6 = 4 .

NOTE: Here –2 is the “minuend” and –6 is the “subtrahend”. The subtrahend is always taken away from the minuend.

Australian Business Number (ABN): 64 011 074 054

http://tutoring.div1.com.au

P.O. Box 1353, Meadowbank, N.S.W. 2114, AUSTRALIA

−	−7	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6	+7
−7															
−6															
−5															
−4															
−3															
−2		4													
−1															
0															
+1															
+2															
+3															
+4															
+5															
+6															
+7															

c) Multiplication

Table for positive quantities

In the following table the number in the top row multiplies the number in the leftmost column. Example highlighted: $6 \times 2 = 12$.

The result of multiplication is called the “product”.

×	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1															
2															
3															
4															
5															
6		12													
7															
8															
9															
10															
11															
12															
13															
14															
15															

Table for positive & negative quantities

In the following table the number in the top row multiplies the number in the leftmost column. Example highlighted: $-2 \times -6 = +12$.

Australian Business Number (ABN): 64 011 074 054
http://tutoring.div1.com.au
P.O. Box 1353, Meadowbank, N.S.W. 2114, AUSTRALIA

×	−7	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6	+7
−7															
−6															
−5															
−4															
−3															
−2		12													
−1															
0															
+1															
+2															
+3															
+4															
+5															
+6															
+7															

d) Powers

Table for positive quantities

In the following table the number in the leftmost column is raised to the power of the number in the top row. Example highlighted: $5^2 = 25$.

NOTE: Here 2 is the "index" and 5 is the "base". The base is always raised to the power specified by the index.

^	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5		25										
6												
7												
8												
9												
10												
11												
12												
13												
14												
15												
16												

The symbol "^" is not formally recognised in mathematics as representing a power. However, it is a common convention in computer applications.

Australian Business Number (ABN):
64 011 074 054

http://tutoring.div1.com.au
P.O. Box 1353, Meadowbank, N.S.W. 2114, AUSTRALIA

Table for positive & negative quantities

In the following table the number in the leftmost column is raised to the power of the number in the top row. Example highlighted: $(-2)^{-3} = 1/(-2)^3 = -1/8$.

^	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6
−6													
−5													
−4													
−3													
−2				−1/8									
−1													
0													
+1													
+2													
+3													
+4													
+5													
+6													

This document was produced on 26 June 2018.

Australian Business Number (ABN):
64 011 074 054

http://tutoring.div1.com.au
P.O. Box 1353, Meadowbank, N.S.W. 2114, AUSTRALIA